

Reminiscing Everyday Mathematics: My Childhood Workbook

Abstract: *Everyday Mathematics* is an elementary math curriculum for Grades Pre-K to 6. Back in 2007, my elementary school used *Everyday Mathematics* (3rd edition), where mathematics spiraled, alternative algorithms are taught, and a wide range of topics are covered. I had great moments from the program, because there were lots of interesting things that broadened my knowledge of mathematics. My favorite thing about this program is that math-modeling routines: *Frames-and-Arrows*, *What's My Rule*, and *Name-Collection Boxes* continue to be useful even in college-level math. In addition, some 5th/6th grade topics have the potential to be cross-compatible with preliminary college courses namely MAT0650 and MAT1190, due to common concepts shared with each other. As of May 31st, 2021, *Everyday Mathematics* is at its fourth edition, where the curriculum underwent significant changes.

Anthony John Regner @ City Tech/Hunter, CUNY

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For this paper, the third edition (2007) of *Everyday Mathematics* is assumed, unless stated otherwise.

Everyday Mathematics is an elementary-level curriculum from grades Pre-K through 6th developed by the University of Chicago School Mathematics Project and published by McGraw Hill Education. This workbook is considered “reform mathematics”, which means EM places emphasis on alternative methods (such as partial sums and lattice multiplication) instead of traditional methods. In addition, *Everyday Mathematics* is a spiral curriculum, where material is exposed multiple times, indicating that it is not necessary to master concepts the first time.

The exposure to different methods allows students to pick their preferred method of solving operations with multiple digits to fit their individual needs. In my experience, using the standard multiplication algorithm is prone to many mistakes. For instance, I can be carrying the wrong amount due to place-value confusion or having to erase/hide my “carrying” values from digit to digit. Using the lattice method, however, makes multi-digit multiplication much easier.

I just have to find each product by multiplying the two numbers corresponding to each row and column. After I find the products, I add all of those values diagonally starting from the bottom-right. One major disadvantage of lattice multiplication is that I have to set up the lattice (or grid) before actually doing the work.

According to the research paper called *Algorithms in Everyday Mathematics* by UCSMP, although standard algorithms are highly effective, no algorithm is considered the “best”. If paper-and-pencil algorithms are still needed in elementary-level curriculum, alternative methods need to be considered. Historical methods (like lattice multiplication) are used as alternative methods. In *Everyday Mathematics*, one alternative method from each operation are identified as a **focus** algorithm, where it is intended to be taught by schools. In this sense, students can either be accustomed to alternative methods taught in *Everyday Mathematics* or stick to traditional methods. For best results, schools shouldn’t force students to use only one specific algorithm, whether it is standard or alternative. In **Figure 1**, I have shown three ways to solve (625×278) and ended up with the same answer (173,750).

I love *Everyday Mathematics* when I was in elementary school (2007 - 2012), and even right now (2020 – present), because this curriculum demonstrates that elementary-level mathematics is more than arithmetic. It covers a wide range of topics such as geometry, statistics, probability, measurement, and even early algebra. In fact, some topics in 5th and 6th grade *Everyday Mathematics* can be cross-compatible with preliminary college courses such as MAT0650, MAT1190, MAT1272 and MAT1372. In fact, some *Everyday Mathematics* tools (math-modeling routines) remain useful in later

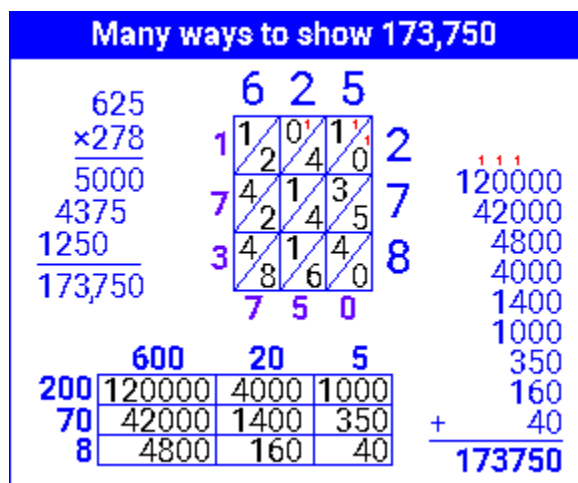


Figure 1 - Many ways to multiply 625×278

college courses such as *Frames-and-Arrows* as sequences in precalculus (MAT1375), *What's My Rule?* as function machines in all calculus courses up to MAT2675, and name-collection boxes as naming equivalent numbers, terms, and expressions in all math courses. *Everyday Mathematics* also allows us to be creative. Back in 2nd grade in September 2008, we had “broken-calculator” problems where we have to express equivalent names for numbers without using certain digits. At that time, while many students were limited to using addition and subtraction, some students also used multiplication and division due to prior knowledge. There were more interesting worksheets we did throughout the EM program, such as “Lesson 7-4: The Wubbles” and “Lesson 10-2: Word Values” in second grade.

Show 100 without using any of these digits: 0, 1, 2, 5

$36 + 64$ $498 - 398$
 $777 \div 7.77$ $93 + 7$
 $\left(\sum_{36}^{39} n - \sum_6^9 n - \sum_6^9 n \right) \div .9$
 $\int_{33}^{38} 3(9) - 7 \, dx$

Figure 2 - Advanced broken calculator problem

However, *Everyday Mathematics* is not without controversy. As soon as the 1st edition of *Everyday Mathematics* was released, this program became subject of nationwide controversy. One of the main reasons that the program became controversial is that it changed fundamental parts of learning math by emphasizing on alternative methods rather than the more standard ones. In addition, some activities favor higher order thinking rather than pure memorization, especially in lower grade levels. As a result, some states completely rejected *Everyday Mathematics* due to content not meeting its state and local standards.

Despite the controversy with the program, I still love *Everyday Mathematics*, because of the good memories I had.

Everyday Mathematics is not a textbook, but rather a workbook. The student components of the workbook are the “Home Links / Study Links”, “Student Math Journals”, “Skills Links”, and the “Student Reference Books”. At the beginning of the school year, a student receives their “Student Math Journal”, and their “Home Links/Study Links” workbook. When a student needs extra practice, they may receive their “Skills Link” book. At times, students need to use their “Student Reference Book” to refer to important concepts learned in class.

On a typical day in a math class, students spend their first few minutes with a “Math Message”. Math Messages are warm-up activities to reinforce their students’ learning. After the “Math Message”, the teacher teaches the lesson. Each lesson in *Everyday Mathematics* usually takes one day, although some lessons can take two days. After teaching, students will work on problems based on the lesson in their Student Math Journals. For homework, students would do “Math Boxes” or “Home Links / Study Links”. “Math Boxes” are mixed practice problems taken from previous material in order to practice maintaining their skills they have already obtained, while “Home Links / Study Links” are additional work they complete to reinforce their understanding of the material they learned from a particular lesson, as well as some practice problems at the bottom. “Math Boxes” work very well, especially when students can’t master concepts

in one exposure. Some students might need multiple exposures to master specific concepts, especially the more difficult ones.

Some teachers may differentiate instruction for students who are performing above or below their grade level. Let's take Grade 6, Lesson 7-8 (Strategies for Multiple-Choice Tests) as an example. The "Enrichment" activity called "Probability and Pascal's Triangle" asks students to find outcomes based on the binomial distribution (that's on Lesson 12 in MAT1372 and Lesson 25 in MAT1375) by observing the patterns they see on Pascal's triangle. On page 240 of *Math Masters*, it tells students to complete Pascal's triangle and explain the patterns they noticed. On the next page, students apply the concepts used in Pascal's triangle to find binomial probabilities of events. For example, I flip eight coins, and two of them ended up landing on HEADS. To find the probability, students refer to Pascal's triangle and they find the probability that 2/8 coins were heads. In this case, it would be $28/2^8 = 28/256 = 10.94\%$. The "Readiness" and "Extra Practice" activities are designed for students who have difficulty grasping concepts in a particular lesson. In this case, students would need to learn the prerequisite material such as writing improper fractions as mixed numbers, and vice versa. Students would also need to visualize the tree diagram to find probabilities of events.

Many math programs do not allow the use of the calculator. However, *Everyday Mathematics* encourages the use of a calculator in many cases. Based on the research, "the use of a calculator can enhance students' performance in number sense, conceptual development, and visualization", according to National Council of Teachers of Mathematics (1998). In the *Everyday Mathematics* program, students use the calculator to recognize place value, create number sequences, and compute very large numbers. In Grades K through 3, students use the TI-108, Casio SL-450, or a similar calculator. In Grades 4 through 6, students use more complex calculators such as the TI-15 and the Casio fx-55. The authors of *Everyday Mathematics* believe that **how** a calculator works is more important than **what** features does a calculator have.

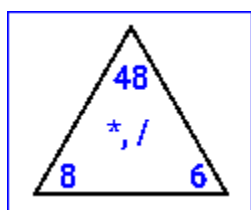


Figure 3 - Fact triangle

Calculator usage is encouraged, but not always. There are instances where the use of a calculator is inappropriate. For example, when students need to practice using algorithms for any operation, calculators should never be used, because they can undermine the learning process. Likewise, calculators are ineffective for solving basic facts, because with fluency of math facts, students can solve them instantaneously and faster than a calculator. Therefore, it is still important for students to recall math facts, with the use of fact triangles.

On the other hand, there are instances where the use of a calculator is important to understand the material, and to apply concepts into real-world situations. One instance would be Home Link 10-4 called *Calculators and Money* (p304 in *Math Masters*) in second grade. In Problem #1, students were asked to put certain amounts on their calculator. If students were to put \$11.50 (or let alone 11.5000) on the calculator, it will truncate the trailing zeros on the right. It shows that 11.5 is numerically

equivalent to \$11.50. If students were to put 95¢ on the calculator, it would show either 95 or 0.95 (the latter is correct) depending on how they interpret the amount. This would give a preliminary exposure to decimals, which would be used in later grades. Another instance of necessary calculator use would be tipping. It is important that students use the calculator to calculate the tip for various services they have. For example, the restaurant bill would be \$84.17 for a family of four eating dinner and desserts. Then, the family chose to tip the waiter 20% for exemplary service. In this case, solving $\$84.17 \times 1.2$ using paper-and-pencil algorithms wouldn't be appropriate, because manual computations are prone to mistakes due to time constraints. Instead of paper-and-pencil algorithms, they would use the calculator, because it is more efficient.

Games are an integral part in *Everyday Mathematics*, because they help reinforce important skills while students enjoy playing them. There are a total of 88 games in grades K-3, and 81 games in grades 4-6 according to their respective Games Correlation Chart. Due to the exhaustive list of games that students can play, it is important to set aside enough time (For example, Fridays should be game days) for students to play games to help them understand relevant concepts. When students enjoy playing EM games, they may continue to play them during their free time while maintaining their skills. One of the games I remembered playing as a child is *Beat the Calculator*. This game challenges the "Brain" to compute basic facts faster than the "Calculator". This shows us that we can compute basic facts mentally, rather than always relying on a calculator. The game *Beat the Calculator* can also be used as an assessment tool to determine the mastery of basic facts for addition and multiplication. For instance, if the "Brain" can compute facts like 9×5 , 6×8 , 8×7 , 4×7 faster than the "Calculator", then the student playing as the "Brain" has demonstrated mastery of basic facts. I enjoyed playing *Beat the Calculator*, because it gave me opportunities to use the calculator when allowed. Many of the EM games use number cards (called the Everything Math Deck), dice and/or spinners as tools to introduce or reinforce the concepts of probability. Therefore, games help students learn concepts related to the lesson in addition to other concepts (like probability), when used appropriately.

The best part in *Everyday Mathematics* is math-modeling routines. These math-modeling routines include *Frames and Arrows*, *What's my Rule?*, *Name-Collection Boxes*, *number lines/grids*, *unit boxes*, *fact families/triangles* and *situation diagrams*. For this paper, I will focus on the first three I listed:

Frames-and-Arrows: This routine is used for identifying numbers in a sequence. The first exposure to *Frames-and-Arrows* is on Lesson 3-8 in 1st grade. Like I mentioned in my National Math Festival post, the *Frames-and-Arrows* introduces the concept of numerical sequences. There are two components in *Frames-and-Arrows* which are

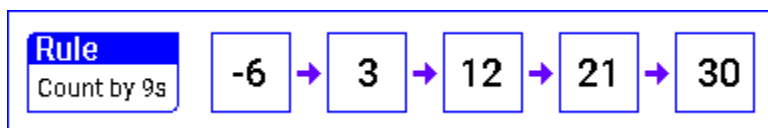


Figure 4 - *Frames-and-Arrows* problem for an arithmetic sequence

called the **rule**, and the **frames**. The **rule** (or common difference/ratio) determines the amount (additive or multiplicative) between each

frame, while the **frames** (terms) represent the current values imposed by the rule. The first set of *Frames-and-Arrows* problems are arithmetic sequences, introduced in 1st

grade. Students either fill in the empty frames or the empty rule given by the numbers. To understand *Frames-and-Arrows*, they need to examine the given sequence (and the rule, if given). Then, they fill out the empty frames by repeatedly adding or subtracting neighboring numbers in each frame. In some cases, students will need to work backwards by finding the inverse of the given rule and filling the frames towards the left. For example, if the rule states “+4” and the rightmost frame is 30, students need to find the opposite of the rule, which is “-4”, and subtract 4 each time: [26, 22, 18, 14, ...]. The second set of *Frames-and-Arrows* problems are geometric sequences, introduced in 2nd grade (Lesson 7-4). It serves a similar purpose to arithmetic sequences; however, students repeatedly multiply or divide neighboring numbers in each frame. One major difference is that corresponding numbers grow or decay

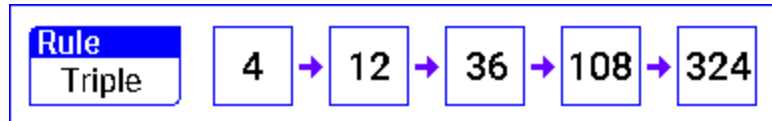


Figure 5 - Frames-and-Arrows problem for a geometric sequence

exponentially, rather than adding/subtracting in a constant rate. In addition, when dividing, the numbers in the sequence converge to zero. The Frames-and-Arrows problems in elementary math help students prepare for more sophisticated sequences in algebra and precalculus (MAT1375). In MAT1375, students were asked to find a specific term using a formula, and even find the sum of the first x terms of the arithmetic sequence and the infinite geometric sequence.

What’s My Rule? is a more complicated variation of the *Frames-and-Arrows* routine. It is a function machine where it accepts inputs (starting values), then modifies the values based on the rule, then outputs the modified values. For example, if I were the put the numbers {1, 3, 5, 7}, into the function machine where the rule is “square, then add one” or “ $x^2 + 1$ ”, then the output of these numbers would be {2, 10, 26, 50}. *What’s My Rule?* allows us to put algebraic functions where expressions can be linear, harmonic, quadratic, cubic, polynomial, exponential, trigonometric, etc. To understand *What’s My Rule?*, students need to examine the “input” numbers, and the rule. Then, they modify the “input” numbers by applying the rule. When “output” numbers are given, students find the inverse of the rule, and apply the inverse to find the “input” numbers. The *What’s My Rule?* routine is introduced in Kindergarten and continues throughout the *Everyday Mathematics* program in all grade levels. It starts as simple as applying one operation (adding or subtracting) on the function machine. As students progress with the *What’s My Rule?* routine, new operations (multiplication and division) can be applied, and then turns the “in” and “out” language into algebraic variables such as x and y , where the rule is written as an equation such as ($y = 2x + 7$) or ($y = x^2 + 4x$). In college-level mathematics, *What’s My Rule?* continues to be useful in college algebra, precalculus, and the entire calculus sequence (until MAT2675). In college, the *What’s My Rule?* routine can be extended to composite functions and multi-variable functions, in terms of $f(g(x))$ or $(f \cdot g)(x)$ and $f(x,y,z)$ respectively. The former is learned in Lesson 6 in MAT1375, while the latter is learned at the very beginning of MAT2675, given the current course outline. In addition, *What’s My Rule?* can also be

	in	out
	1	2
	3	10
	$\sqrt{24}$	25
	i	0

Figure 6 - What's My Rule? table

used to find limits (of both sides) of a given function, either towards a certain number, or towards infinity in MAT1475.

Name-collection boxes store equivalent names for each number, term, or expression, since there are infinitely many ways to express a certain number. It is first introduced in first grade on Lesson 6-2 (Equivalent Names). The way to express equivalent names for numbers is the use of different operations (+, -, ×, ÷), tallies, arrays, concrete objects, dice, dominoes, money, and numbers written in words (including different languages). The use of *name-collection boxes* continues throughout the program. In higher grades (starting in 4th grade), pictorial representations of the number are retired in favor of numerical expressions, because of the way the *name-collection boxes* are formatted.

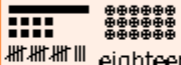
0	-1 + 1	1	2 - 1	18	3 * 6	-4	-(4)	π
0(19)	1(0)	$x * \frac{1}{x}$	$\sin(\pi/2)$	17 + 1	19 - 1	5 - 9	$\frac{16}{-4}$	3.1415926...
6 - 6	$\ln(1)$	$\frac{1}{2} + \frac{1}{2}$	$\cos(0)$	100	$\frac{108}{6}$	$0 - (2 * 2)$		circumference of circle times half
$\sin(0)$	0 / x	$\frac{1}{2} + \frac{1}{2}$	$\cot(\pi/4)$	$-\frac{82}{6}$		$(11 - 7) * -1$		$\sin^4(0)$ $\cos^4(-1)$
$\tan(0)$	0 * x	8 / 8	$\sec(0)$		eighteen	-6 + 2	four less than zero	$\approx \frac{22}{7}$ or $3\frac{1}{7}$
$\int_9^9 x dx$	zero	$\frac{x}{x}$	e^0	dieciocho, dezoito		negative four		
$(-5 * 2 + 7) + 3$	cero	1	uno, um	$\sum_2^4 2n$	$36 * \frac{1}{2}$	$2(-1 + -1)$	$\frac{96}{-100}$	
		$\int_0^1 9x^8 dx$						

Figure 7 - Name-collection boxes for five selected numbers

However, in college-level math (algebra, calculus, discrete math, linear algebra, and differential equations), the use of *name-collection boxes* is **enhanced** as students can collect more names for terms beyond numbers. Further uses of *name-collection boxes* include polar/cylindrical/spherical coordinates, indeterminate forms, differentiable functions, non-real expressions, trigonometric substitution forms, invertible matrices, identity matrices, and lots more! Using the extended definition of *name-collection boxes*, these are used to collect examples that fit into that specific category, that has the same functionality as folders. In addition, they are great for organizing examples in general.

Side note: Surprisingly in WeBWork, the system accepts *any equivalent name* of the correct answer, rather than just the correct answer itself. When a problem in WeBWork doesn't ask us to simplify our answer, we can express the correct answer in infinitely many different ways, such that $3/2$ is also equal to $1\frac{1}{2}$, 1.5, $1 + (1/2)$, or we can go as wild as $3(1/2)$ or $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$. **Because of this convention, I am reminding the WeBWork team of this!**

This paper won't be complete without sample worksheets of *Everyday Mathematics*! I will be including eight sample worksheets at the end of this paper for you to take a closer look.

- The first page “Using Factor Trees” shows the Fundamental Theorem of Arithmetic. Students are asked to make factor trees for each number and write the prime factorization. (In MAT2440 and MAT3021)
- The second page “Probability Investigations” uses the Counting Principle (combinatorics) to find the number of possible combinations in given situations. (In MAT2540 and Lesson 7 in MAT1372)
- The third page “Range, Median, and Mean” reinforces the concept of measures of central tendency. Students are also asked to make stem-and-leaf plots. (Lessons 2 and 3 in MAT1372)
- The fourth and fifth pages “Frequency Tables and Venn Diagrams” shows conditional probability. Students are asked to find probabilities using tree and Venn diagrams. (Lesson 8 in MAT1372)
- The sixth page “Pascal’s Triangle” informally introduces the Binomial Theorem. (Lesson 25 in MAT1375)
- The seventh page “Probability and Pascal’s Triangle” reinforces Pascal’s Triangle to find probabilities via binomial distribution. This also briefly reviews combinatorics. (Lesson 12 in MAT1372)
- The eighth page “Solving Equations by Trial and Error” introduces quadratic equations. Students are asked to find one solution of the equation through trial and error. In later grades, they can factor the quadratic equation directly, use the quadratic formula, and use the square root property. (Lesson 11 in MAT1275)

These worksheets I chose have the potential to be compatible with preliminary college math courses. In case of statistics and probability topics, they also have the potential to be compatible with MAT1272 / MAT1372.

Just after I graduated (in June 2012) from elementary school, *Everyday Mathematics* revised the third edition, so it can align with the new common core state standards (CCSS). While the curriculum remains relatively unchanged, some lessons were added, modified, tweaked, or removed in response to these new CCSS. In 2014, *Everyday Mathematics* released the 4th edition, which overhauled the curriculum. This edition came with lots of changes. As opposed to 10-12 units per grade level, there is now only 8-9 units per grade level. In addition, there are two 2-day lessons in each unit. In the middle of each unit, it features open response questions, which promotes critical thinking and problem-solving skills. At the end of each unit, there is a periodic assessment. In Day 2 of these assessments, they alternate between cumulative assessments and open-response questions. Also introduced in the 4th edition, there are quick-look cards in which they would be projected (or shown) for only a few seconds, and activity cards that are shown as posters used in differentiation (readiness, enrichment, extra practice) to address individual student needs.

It has been nine years since the last time I used *Everyday Mathematics* in school. In June 2020, due to great memories I made in this program, I have become a fan of *Everyday Mathematics* program once again. To further support the curriculum, I bought these Skills Links books (3rd edition) as souvenirs.

Works cited

“The Role of Calculators in Everyday Mathematics.” Everyday Mathematics, The University of Chicago School Mathematics Project, everydaymath.uchicago.edu/about/understanding-em/calculators/.

“The Role of Games in Everyday Mathematics.” Everyday Mathematics, The University of Chicago School Mathematics Project, everydaymath.uchicago.edu/about/understanding-em/games/.

Further reading

[Home page] <https://everydaymath.uchicago.edu/>

[Research – Algorithms] <https://everydaymath.uchicago.edu/about/research-results/algorithms.pdf>

[K-3 Games Correlation Chart]
https://everydaymath.uchicago.edu/teachers/games_correlation_k-3_ccss.pdf

[4-6 Games Correlation Chart]
https://everydaymath.uchicago.edu/teachers/games_correlation_4-6_ccss.pdf

Related resources

[Broken calculator] <https://www.mathsisfun.com/games/broken-calculator.html>

[Everyday Mathematics eToolkit] https://connected.mcgraw-hill.com/etool/_etoolkit/2.1.0/index.html